

# Reconstructing holographic quintessence

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The holographic dark energy model is an attempt for probing the nature of dark energy within the framework of quantum gravity. The dimensionless parameter  $c$  determines the main property of the holographic dark energy. With the choice of  $c \geq 1$ , the holographic dark energy can be described completely by a quintessence scalar field. In this paper, we show this quintessential description of the holographic dark energy with  $c \geq 1$  and reconstruct the potential of the quintessence as well as the dynamics of the scalar field.

It has been confirmed admittedly that our universe is experiencing an accelerating expansion at the present time, by many cosmological experiments, such as observations of large scale structure (LSS) [1], searches for type Ia supernovae (SNIa) [2], and measurements of the cosmic microwave background (CMB) anisotropy [3]. This cosmic acceleration observed strongly supports the existence of a mysterious exotic matter, dark energy, with large enough negative pressure, whose energy density has been a dominative power of the universe. The astrophysical feature of dark energy is that it remains unclustered at all scales where gravitational clustering of baryons and nonbaryonic cold dark matter can be seen. Its gravity effect is shown as a repulsive force so as to make the expansion of the universe accelerate when its energy density becomes dominative power of the universe. The combined analysis of cosmological observations suggests that the universe is spatially flat, and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although we can affirm that the ultimate fate of the universe is determined by the feature of dark energy, the nature of dark energy as well as its cosmological origin remain enigmatic at present. However, we still can propose some candidates to interpret or describe the properties of dark energy. The most obvious theoretical candidate of dark energy is the cosmological constant  $\lambda$  (vacuum energy) [4, 5] which has the equation of state  $w = -1$ . However, as is well known, there are two difficulties arise from the cosmological constant scenario, namely the two famous cosmological constant problems — the “fine-tuning” problem and the “cosmic coincidence” problem [6]. The fine-tuning problem asks why the vacuum energy density today is so small compared to typical particle scales. The vacuum energy density is of order  $10^{-47}\text{GeV}^4$ , which appears to require the introduction of a new mass scale 14 or so orders of magnitude smaller than the electroweak scale. The second difficulty, the cosmic coincidence problem, says: Since the energy densities of vacuum energy and dark matter scale so differently during the expansion history of the universe, why are they nearly equal today? To get this coincidence, it appears that their ratio must be set to a specific, infinitesimal value in the very early universe.

Theorists have made lots of efforts to try to resolve the cosmological constant problem, but all these efforts were turned out to be unsuccessful. Of course the theoretical consideration is still in process and has made some progresses. In recent years, many string theorists have devoted to understand and shed light on the cosmological constant or dark energy within the string framework. The famous Kachru-Kalosh-Linde-Trivedi (KKLT) model [7] is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Furthermore, string landscape idea [8] has been proposed for shedding light on the cosmological constant problem based upon the anthropic principle and multiverse speculation. However, there remain other candidates to explaining dark energy.

An alternative proposal for dark energy is the dynamical dark energy scenario. The cosmological constant puzzles may be better interpreted by assuming that the vacuum energy is canceled to exactly zero by some unknown mechanism and introducing a dark energy component with a dynamically variable equation of state. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential. Actually, this mechanism is enlightened to a great extent by the inflationary cosmology. As we have known, the occurrence of the current accelerating expansion of the universe is not the first time in the expansion history of the universe. There is significant observational evidence strongly supporting that the universe underwent an early inflationary epoch, over sufficiently small time scales, during which its expansion rapidly accelerated under the driven of an “inflaton” field which had properties similar to those of a cosmological constant. The inflaton field, to some extent, can be viewed as a kind of dynamically evolving dark energy. Hence, the scalar field models involving a minimally coupled scalar field are proposed, inspired by inflationary cosmology, to construct dynamically evolving models of dark energy. The only difference between the dynamical scalar-field dark energy and the inflaton is the energy scale they possess. Famous examples of scalar-field dark energy models include quintessence [9],

$K$ -essence [10], tachyon [11], phantom [12], ghost condensate [13, 14] and quintom [15], and so forth. Generically, there are two points of view on the scalar-field models of dynamical dark energy. One viewpoint regards the scalar field as a fundamental field of the nature. The nature of dark energy is, according to this viewpoint, completely attributed to some fundamental scalar field which is omnipresent in supersymmetric field theories and in string/M theory. The other viewpoint supports that the scalar field model is an effective description of an underlying theory of dark energy. On the whole, it seems that the latter is the mainstream point of view. Since we regard the scalar field model as an effective description of an underlying theory of dark energy, a question arises asking: What is the underlying theory of the dark energy? Of course, hitherto, this question is far beyond our present knowledge, because that we can not entirely understand the nature of dark energy before a complete theory of quantum gravity is established. However, although we are lacking a quantum gravity theory today, we still can make some attempts to probe the nature of dark energy according to some principles of quantum gravity. The holographic dark energy model is just an appropriate example, which is constructed in the light of the holographic principle of quantum gravity theory. That is to say, the holographic dark energy model possesses some significant features of an underlying theory of dark energy.

The distinctive feature of the cosmological constant or vacuum energy is that its equation of state is always exactly equal to  $-1$ . However, when considering the requirement of the holographic principle originating from the quantum gravity speculation, the vacuum energy will become dynamically evolving dark energy. Actually, the dark energy problem may be in principle a problem belongs to quantum gravity [16]. In the classical gravity theory, one can always introduce a cosmological constant to make the dark energy density be an arbitrary value. However, a complete theory of quantum gravity should be capable of making the property of dark energy, such as the energy density and the equation of state, be determined definitely and uniquely [16]. Currently, an interesting attempt for probing the nature of dark energy within the framework of quantum gravity is the so-called “holographic dark energy” proposal [17, 18, 19, 20]. It is well known that the holographic principle is an important result of the recent researches for exploring the quantum gravity (or string theory) [21]. This principle is enlightened by investigations of the quantum property of black holes. Roughly speaking, in a quantum gravity system, the conventional local quantum field theory will break down. The reason is rather simple: For a quantum gravity system, the conventional local quantum field theory contains too many degrees of freedom, and so many degrees of freedom will lead to the formation of black hole so as to break down the effectiveness of the quantum field theory.

For an effective field theory in a box of size  $L$ , with

UV cut-off  $\Lambda$  the entropy  $S$  scales extensively,  $S \sim L^3 \Lambda^3$ . However, the peculiar thermodynamics of black hole [22] has led Bekenstein to postulate that the maximum entropy in a box of volume  $L^3$  behaves nonextensively, growing only as the area of the box, i.e. there is a so-called Bekenstein entropy bound,  $S \leq S_{BH} \equiv \pi M_{Pl}^2 L^2$ . This nonextensive scaling suggests that quantum field theory breaks down in large volume. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, Cohen et al. [17] proposed a more restrictive bound – the energy bound. They pointed out that in quantum field theory a short distance (UV) cut-off is related to a long distance (IR) cut-off due to the limit set by forming a black hole. In other words, if the quantum zero-point energy density  $\rho_{de}$  is relevant to a UV cut-off, the total energy of the whole system with size  $L$  should not exceed the mass of a black hole of the same size, thus we have  $L^3 \rho_{de} \leq L M_{Pl}^2$ . This means that the maximum entropy is in order of  $S_{BH}^{3/4}$ . When we take the whole universe into account, the vacuum energy related to this holographic principle [21] is viewed as dark energy, usually dubbed holographic dark energy. The largest IR cut-off  $L$  is chosen by saturating the inequality so that we get the holographic dark energy density

$$\rho_{de} = 3c^2 M_{Pl}^2 L^{-2}, \quad (1)$$

where  $c$  is a numerical constant, and  $M_{Pl} \equiv 1/\sqrt{8\pi G}$  is the reduced Planck mass. If we take  $L$  as the size of the current universe, for instance the Hubble scale  $H^{-1}$ , then the dark energy density will be close to the observational result. However, Hsu [19] pointed out that this yields a wrong equation of state for dark energy. Li [20] subsequently proposed that the IR cut-off  $L$  should be taken as the size of the future event horizon

$$R_{eh}(a) = a \int_t^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{Ha'^2}. \quad (2)$$

Then the problem can be solved nicely and the holographic dark energy model can thus be constructed successfully. The holographic dark energy scenario may provide simultaneously natural solutions to both dark energy problems as demonstrated in Ref.[20]. The holographic dark energy model has been tested and constrained by various astronomical observations [23, 24, 25]. For other extensive studies, see e.g. [26].

Consider now a spatially flat FRW (Friedmann-Robertson-Walker) universe with matter component  $\rho_m$  (including both baryon matter and cold dark matter) and holographic dark energy component  $\rho_{de}$ , the Friedmann equation reads

$$3M_{Pl}^2 H^2 = \rho_m + \rho_{de}, \quad (3)$$

or equivalently,

$$E(z) \equiv \frac{H(z)}{H_0} = \left( \frac{\Omega_{m0}(1+z)^3}{1 - \Omega_{de}} \right)^{1/2}, \quad (4)$$

where  $z = (1/a) - 1$  is the redshift of the universe. Note that we always assume spatial flatness throughout this paper as motivated by inflation. Combining the definition of the holographic dark energy (1) and the definition of the future event horizon (2), we derive

$$\int_a^\infty \frac{d \ln a'}{H a'} = \frac{c}{H a \sqrt{\Omega_{\text{de}}}}. \quad (5)$$

We notice that the Friedmann equation (4) implies

$$\frac{1}{H a} = \sqrt{a(1 - \Omega_{\text{de}})} \frac{1}{H_0 \sqrt{\Omega_{\text{m}0}}}. \quad (6)$$

Substituting (6) into (5), one obtains the following equation

$$\int_x^\infty e^{x'/2} \sqrt{1 - \Omega_{\text{de}}} dx' = c e^{x/2} \sqrt{\frac{1}{\Omega_{\text{de}}} - 1}, \quad (7)$$

where  $x = \ln a$ . Then taking derivative with respect to  $x$  in both sides of the above relation, we get easily the dynamics satisfied by the dark energy, i.e. the differential equation about the fractional density of dark energy,

$$\Omega'_{\text{de}} = -(1+z)^{-1} \Omega_{\text{de}} (1 - \Omega_{\text{de}}) \left( 1 + \frac{2}{c} \sqrt{\Omega_{\text{de}}} \right), \quad (8)$$

where the prime denotes the derivative with respect to the redshift  $z$ . This equation describes behavior of the holographic dark energy completely, and it can be solved exactly [20]. From the energy conservation equation of the dark energy, the equation of state of the dark energy can be given [20]

$$w = -1 - \frac{1}{3} \frac{d \ln \rho_{\text{de}}}{d \ln a} = -\frac{1}{3} \left( 1 + \frac{2}{c} \sqrt{\Omega_{\text{de}}} \right). \quad (9)$$

Note that the formula  $\rho_{\text{de}} = \frac{\Omega_{\text{de}}}{1 - \Omega_{\text{de}}} \rho_{\text{m}}^0 a^{-3}$  and the differential equation of  $\Omega_{\text{de}}$  (8) are used in the second equal sign. It can be seen clearly that the equation of state of the holographic dark energy evolves dynamically and satisfies  $-(1 + 2/c)/3 \leq w \leq -1/3$  due to  $0 \leq \Omega_{\text{de}} \leq 1$ . Hence, we see clearly that when taking the holographic principle into account the vacuum energy becomes dynamically evolving dark energy. The parameter  $c$  plays a significant role in this model. If one takes  $c = 1$ , the behavior of the holographic dark energy will be more and more like a cosmological constant with the expansion of the universe, such that ultimately the universe will enter the de Sitter phase in the far future. As is shown in [20], if one puts the parameter  $\Omega_{\text{de}}^0 = 0.73$  into (9), then a definite prediction of this model,  $w_0 = -0.903$ , will be given. On the other hand, if  $c < 1$ , the holographic dark energy will exhibit appealing behavior that the equation of state crosses the “cosmological-constant boundary” (or “phantom divide”)  $w = -1$  during the evolution. This kind of dark energy is referred to as “quintom” [15] which is slightly favored by current observations [28, 29]. For extensive studies on quintom model see e.g. [27]. If  $c > 1$ ,

the equation of state of dark energy will be always larger than  $-1$  such that the universe avoids entering the de Sitter phase and the Big Rip phase. Hence, we see explicitly, the value of  $c$  is very important for the holographic dark energy model, which determines the feature of the holographic dark energy as well as the ultimate fate of the universe. As an illustrative example, we plot in figure 1 the selected evolutions of the equation of state of holographic dark energy. We show in the plot the cases of  $c = 1.0, 1.1, 1.2$  and  $0.9$ . It is clear to see that the cases in  $c \geq 1$  always evolve in the region of  $w \geq -1$ , whereas the case of  $c < 1$  behaves as a quintom whose equation of state  $w$  crosses the cosmological constant boundary  $-1$  during the evolution.

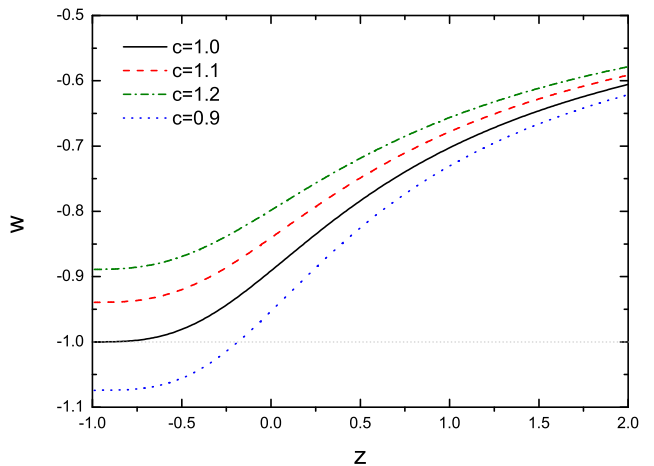


FIG. 1: The evolutions of the equation of state of holographic dark energy. Here we take  $\Omega_{\text{m}0} = 0.3$ , and show the cases for  $c = 1.0, 1.1, 1.2$  and  $0.9$ . Clearly, the cases in  $c \geq 1$  behave as holographic quintessence, and the case of  $c < 1$  behaves as holographic quintom.

As has been analyzed above, the holographic dark energy scenario reveals the dynamical nature of the vacuum energy. When taking the holographic principle into account, the vacuum energy density will evolve dynamically. On the other hand, as has already mentioned, the scalar field dark energy models are often viewed as effective description of the underlying theory of dark energy. However, the underlying theory of dark energy can not be achieved before a complete theory of quantum gravity is established. We can, nevertheless, speculate on the underlying theory of dark energy by taking some principles of quantum gravity into account. The holographic dark energy model is no doubt a tentative in this way. We are now interested in that if we assume the holographic vacuum energy scenario as the underlying theory of dark energy, how the scalar field model can be used to effectively describe it.

The quintessence scalar field  $\phi$  evolves in its potential  $V(\phi)$  and seeks to roll towards the minimum of the potential, according to the Klein-Gordon equation  $\ddot{\phi} + 3H\dot{\phi} = -dV/d\phi$ . The rate of evolution is driven by

the slope of the potential and damped by the cosmic expansion through the Hubble parameter  $H$ . The energy density and pressure are  $\rho_\phi = \dot{\phi}^2/2 + V$ ,  $p_\phi = \dot{\phi}^2/2 - V$ , so that the equation of state of quintessence  $w_\phi = p_\phi/\rho_\phi$  evolves in a region of  $-1 < w_\phi < 1$ . Usually, for making the universe's expansion accelerate, it should be required that  $w_\phi$  must satisfy  $w_\phi < -1/3$ . Nevertheless, it can be seen clearly that the quintessence scalar field can not realize the equation of state crossing  $-1$  [30]. Therefore, only the holographic dark energy in cases of  $c \geq 1$  can be described by the quintessence.

Now let us see the constraint results for the holographic dark energy model from the observational data. When combining the information from SNIa [31], CMB [3] and LSS [32], the fitting for the holographic dark energy model gives the parameter constraints in  $1\sigma$ :  $c = 0.81^{+0.23}_{-0.16}$ ,  $\Omega_{m0} = 0.28 \pm 0.03$ , with  $\chi^2_{\min} = 176.67$  [23]. In this joint analysis, the SNIa data come from the 157 “gold” data [31] including 14 high redshift data from the *Hubble Space Telescope* (HST)/Great Observatories Origins Deep Survey (GOODS) program and previous data, the CMB information comes from the measured value of the CMB shift parameter  $R$  given by [3]  $R \equiv \Omega_{m0}^{1/2} \int_0^{z_{\text{CMB}}} dz'/E(z') = 1.716 \pm 0.062$ , where  $z_{\text{CMB}} = 1089$  is the redshift of recombination, and the LSS information is provided by the baryon acoustic oscillation (BAO) measurement [32]  $A \equiv \Omega_{m0}^{1/2} E(z_{\text{BAO}})^{-1/3} [(1/z_{\text{BAO}}) \int_0^{z_{\text{BAO}}} dz'/E(z')]^{2/3} = 0.469 \pm 0.017$ , where  $z_{\text{BAO}} = 0.35$ . Furthermore, the X-ray gas mass fraction of rich clusters, as a function of redshift, has also been used to constrain the holographic dark energy model [24]. The  $f_{\text{gas}}$  values are provided by *Chandra* observational data, the X-ray gas mass fraction of 26 rich clusters, released by Allen et al. [33]. The main results, i.e. the  $1\sigma$  fit values for  $c$  and  $\Omega_{m0}$  are:  $c = 0.61^{+0.45}_{-0.21}$  and  $\Omega_{m0} = 0.24^{+0.06}_{-0.05}$ , with the best-fit chi-square  $\chi^2_{\min} = 25.00$  [24]. We see that, basically, in one-sigma error range, the holographic dark energy will behave as quintom-like dark energy whose equation-of-state crosses the  $w = -1$  line during the evolution.<sup>1</sup>

On the other hand, even though the current observational data indicate that the parameter  $c$  in the holographic model seems smaller than 1, the possibility of  $c \geq 1$  can not be ruled out yet. For example, for the

upper limit of the error one sigma,  $c = 1.04$  in the result of the joint analysis of SNIa+CMB+LSS;  $c = 1.06$  in the result of the analysis of X-ray gas data. In particular, it must be pointed out that the choice of  $c < 1$ , on theoretical level, will bring some troubles. The cases of  $c < 1$  will lead to dark energy behaving as a phantom eventually, which violates the weak energy condition of general relativity,<sup>2</sup> and the Gibbons-Hawking entropy will thus decrease since the event horizon shrinks, which violates the second law of thermodynamics as well. Besides, the quantum instability may often be encountered in quintom models when the  $w = -1$  crossing happens. What is more, when the future event horizon as the IR cut-off becomes shorter than the UV cut-off within a finite time in the future, the definition of the holographic dark energy will break down. Consequently, from a theoretical viewpoint, the choice of  $c \geq 1$ , especially of  $c = 1$ , is more appropriate. For the favor of  $c < 1$  from the currently available observational data, a possible interpretation says that this maybe a gloss due to lack of sufficiently precise data. Anyway, the holographic dark energy with  $c \geq 1$ , especially with  $c = 1$ , seems reasonable from theoretical viewpoint. One the whole, since the data analysis can not rule out the possibility of  $c \geq 1$  completely, the cases of  $c \geq 1$  are worth investigating in detail. We can establish a correspondence between the holographic dark energy with  $c \geq 1$  and quintessence scalar field, and describe holographic dark energy in this case effectively by making use of quintessence. We refer to this case as “holographic quintessence”.

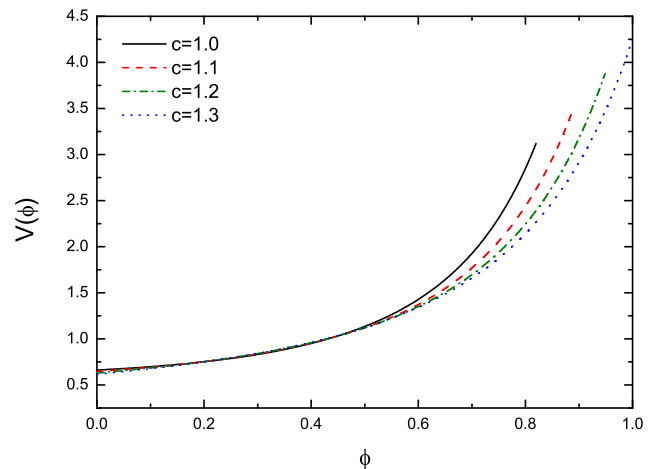


FIG. 2: The reconstruction of the potential for the holographic quintessence, where  $\phi$  is in unit of  $M_{\text{Pl}}$  and  $V(\phi)$  in  $\rho_{\text{co}}$ . We take here  $\Omega_{m0} = 0.3$ .

<sup>1</sup> However, the analysis of the latest observational data shows that this conclusion is somewhat changed, see [34] for details. In this paper, the authors derive constraints on the holographic dark energy model from the latest observational data including the gold sample of 182 SNIa, the CMB shift parameter given by the 3-year WMAP observations, and the BAO measurement from the SDSS. The joint analysis gives the fit results in  $1\sigma$ :  $c = 0.91^{+0.26}_{-0.18}$  and  $\Omega_{m0} = 0.29 \pm 0.03$ . That is to say, though the possibility of  $c < 1$  is more favored, the possibility of  $c > 1$  can not be excluded in one-sigma error range. So, according to the new data, the evidence for the quintom feature in the holographic dark energy model is not as strong as before.

<sup>2</sup> It is remarkable that the phantom behavior of  $w < -1$  also violates the null energy condition, which has been significant subject of investigations, see [35] for example. In [35], the authors show that violation of the null energy condition implies instability in a broad class of models, which indicates for dark energy that  $w$  is unlikely to be less than  $-1$ .

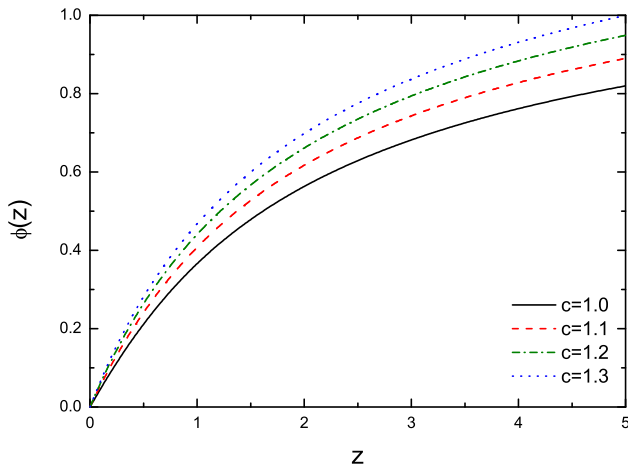


FIG. 3: The revolutions of the scalar-field  $\phi(z)$  for the holographic quintessence, where  $\phi$  is in unit of  $M_{\text{Pl}}$ . We take here  $\Omega_{\text{m}0} = 0.3$ .

The quintessence potential  $V(\phi)$  can be reconstructed from supernova observational data [36, 37]. In addition, from some specific parametrization forms of the equation of state  $w(z)$ , one can also reconstruct the quintessence potential  $V(\phi)$  [38]. The reconstruction method can also be generalized to scalar-tensor theories [39],  $f(R)$  gravity [40], a dark energy fluid with viscosity terms [41], and also the generalized ghost condensate model [42]. For a reconstruction program for a very general scalar-field Lagrangian density see [43]. As discussed above, the holographic dark energy possesses some significant features of the quantum gravity theory. So, to some extent, we can regard the holographic vacuum energy scenario as an underlying theory of dark energy. The scalar-field dark energy model then can be considered as an effective description of this holographic theory. When  $c < 1$  in the holographic scenario, the quintom-like behavior will occur, and we refer to this case as “holographic quintom”. The reconstruction of the scalar-field model (the generalized ghost condensate model) according to the holographic quintom has been investigated in detail in [44]. Now we are focussing on the reconstruction of the holographic quintessence. We shall reconstruct the quintessence potential and the dynamics of the scalar field in the light of the holographic dark energy with  $c \geq 1$ . According to the forms of quintessence energy density and pressure, one can easily derive the scalar potential and kinetic energy term as

$$\frac{V(\phi)}{\rho_{c0}} = \frac{1}{2}(1 - w_\phi)\Omega_\phi E^2, \quad (10)$$

$$\frac{\dot{\phi}^2}{\rho_{c0}} = (1 + w_\phi)\Omega_\phi E^2, \quad (11)$$

where  $\rho_{c0} = 3M_{\text{Pl}}^2 H_0^2$  is today’s critical density of the universe. If we establish the correspondence between the

holographic dark energy with  $c \geq 1$  and quintessence scalar field, then  $E$ ,  $\Omega_\phi$  and  $w_\phi$  are given by Eqs. (4), (8) and (9). Furthermore, the derivative of the scalar field  $\phi$  with respect to the redshift  $z$  can be given

$$\frac{\phi'}{M_{\text{Pl}}} = \pm \frac{\sqrt{3(1 + w_\phi)\Omega_\phi}}{1 + z}, \quad (12)$$

where the sign is actually arbitrary since it can be changed by a redefinition of the field,  $\phi \rightarrow -\phi$ . Consequently, we can easily obtain the evolutionary form of the field

$$\phi(z) = \int_0^z \phi' dz, \quad (13)$$

by fixing the field amplitude at the present epoch ( $z = 0$ ) to be zero,  $\phi(0) = 0$ .

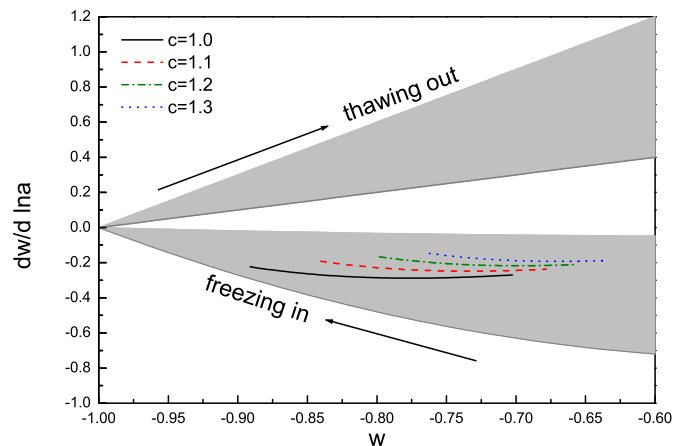


FIG. 4: The evolutionary trajectories of the holographic quintessence in  $w - dw/d \ln a$  phase space. The shaded regions are occupied by thawing and freezing models respectively. The arrows denote the evolutionary directions. The leftmost point of the trajectories corresponds to the present; the rightmost point is at  $z = 1$ . As the same in previous figures, the present fractional matter density is taken as  $\Omega_{\text{m}0} = 0.3$ .

The reconstructed quintessence potential  $V(\phi)$  is plotted in figure 2, where  $\phi(z)$  is also reconstructed according to Eqs. (12) and (13), also displayed in figure 3. Selected curves are plotted for the cases of  $c = 1.0, 1.1, 1.2$  and  $1.3$ , and the present fractional matter density is chosen to be  $\Omega_{\text{m}0} = 0.3$ . From figures 2 and 3, we can see the dynamics of the scalar field explicitly. Obviously, the scalar field  $\phi$  rolls down the potential with the kinetic energy  $\dot{\phi}^2$  gradually decreasing. The equation of state of the quintessence  $w_\phi$ , accordingly, decreases gradually with the cosmic evolution, and as a result  $dw_\phi/d \ln a < 0$ . As suggested in [45], quintessence models can be divided into two classes, “thawing” models and “freezing” models. Thawing models depict those scalar fields that evolve from  $w \approx -1$  but grow less negative with time as  $dw/d \ln a > 0$ ; freezing models, whereas, describe those fields evolve from

$w > -1$ ,  $dw/d\ln a < 0$  to  $w \rightarrow -1$ ,  $dw/d\ln a \rightarrow 0$ . Roughly, the holographic quintessence should be ascribed to the freezing model. Figure 4 illustrates the freezing behavior of the holographic quintessence. Note that it has been indicated in [45] that a practical limit of applicability for thawing and freezing bounds should be  $w \lesssim -0.8$ . Since here we only want to show the freezing behavior of holographic quintessence in the  $w - dw/\ln a$  phase space, the applicability of these regions are continued to  $w \lesssim -0.6$ . As we have seen, the dynamics of the holographic quintessence can be explored explicitly by the reconstruction.

The scalar-field models of dark energy can be viewed as low-energy effective description of the underlying theory (e.g. quantum gravity theory). The quintessence discussed in this paper is specified to an ordinary scalar field minimally coupled to gravity, namely the canonical scalar field. It is remarkable that the resulting model with the reconstructed potential is the unique canonical single-scalar model that can reproduce the holographic evolution with  $c \geq 1$  of the universe. Of course, the aforementioned discussion can be easily generalized to other non-canonical scalar fields, such as  $K$ -essence and tachyon. Moreover, it should be noted that the holographic evolution with  $c < 1$  can also be reproduced by a non-canonical scalar-field (generalized ghost condensate), see [44] for details.

In conclusion, we suggest in this paper a correspondence between the holographic dark energy scenario and the quintessence scalar-field model. We adopt the viewpoint of that the scalar field models of dark energy are effective theories of an underlying theory of dark energy. The underlying theory, though has not been achieved

presently, is presumed to possess some features of a quantum gravity theory, which can be explored speculatively by taking into account the holographic principle of quantum gravity theory. Consequently, the vacuum energy acquires the dynamical property when imposing the holographic principle. Though the currently available observational data imply that the holographic dark energy more likely resembles a quintom, i.e.  $w$  crosses  $-1$ , the data analysis does not rule out the possibility of  $w > -1$  yet. Moreover, the model of  $w > -1$  can avoid some troubles the model of  $w = -1$  crossing encounters. If we regard the scalar-field model (such as quintessence) as an effective description of such a theory (holographic vacuum energy), we should be capable of using the scalar-field model to mimic the evolving behavior of the dynamical vacuum energy and reconstructing this scalar-field model according to the evolutionary behavior of holographic dark energy. We show that the holographic dark energy with  $c \geq 1$  can be described totally by the quintessence in a certain way. A correspondence between the holographic dark energy and quintessence has been established, and the potential of the holographic quintessence and the dynamics of the field have been reconstructed.

#### Acknowledgements

The author would like to thank Miao Li and Feng-Quan Wu for useful discussions. He is also grateful to Shinji Tsujikawa and Alexander Vikman for helpful correspondences. This work was supported in part by the Natural Science Foundation of China.

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